

THE 2ND INTERNATIONAL WORKSHOP ON CRITICAL BEHAVIOR IN LATTICE MODELS



APRIL 5-7, 2018
ANQING, CHINA

ANQING NORMAL UNIVERSITY

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Activities

1. Opening

Time: 8:15-8:25, April 5

2. Banquet

Time: 18:00-20:00, April 6

The banquet is to celebrate the 75th birthday of Prof. Henk W. J. Blöte

3. Tour

Time: April 8-9

Excursion to Yellow Mountain (self-paid)

Program

5 th April	
8:15-8:25	Opening
8:25-8:45	Photo
Topic: Henk/student/student of student	
Chair: Murray Batchelor (Chongqing University, China)	
8:45-9:15	<i>The zoo of two-dimensional $O(n)$ models</i> Henk W. J. Bloete (Leiden University, Netherlands)
9:15-9:45	<i>From Critical Phenomena to Colloids: Phase separation out of equilibrium</i> Erik Luijten (Northwestern University, USA)
9:45-10:15	<i>The halon: a quasiparticle featuring critical charge fractionalization</i> Kun Chen (陈锟 Rutgers University, USA)
10:15-10:35	Tea Break
Topic: Classical lattice models	
Chair: Hai-Jun Zhou (周海军 Institute of Theoretical Physics, CAS)	
10:35-11:05	<i>Critical percolation in anisotropic systems</i> Robert Ziff (University of Michigan, USA)
11:05-11:35	<i>Behavior of the specific heat in the Ising model for $d \geq 4$</i> Per-Håkan Lundow (Umea University, Sweden)
11:35-12:05	<i>Finite- Size Scaling of Correlation Function</i> Xiao-Song Chen (陈晓松 Institute of Theoretical Physics, Chinese Academy of Sciences)
12:05-12:25	<i>Conformal Bootstrap Approach to Critical Phenomena</i> Ning Su (苏宁 Institute of Theoretical Physics, Chinese Academy of Sciences)
12:25-14:00	Lunch Break
Topic: Soft matter physics	
Chair: Bo Zheng (郑波 Zhejiang University)	
14:00-14:30	<i>Supercritical phenomenon in complex liquids</i> Li-Mei Xu (徐莉梅 Peking University)
14:30-15:00	<i>Enhanced sampling and metastable state analyzing in soft matters</i> Xin Zhou (周昕 University of Chinese Academy of Science)
15:00-15:20	<i>Entropy Stabilizes Floppy Crystals of Mobile DNA-Coated Colloids</i> Hao Hu (胡皓 Nanyang Technological University of Singapore)
15:20-15:40	<i>Phase transitions in a confined polyelectrolyte solution</i> Bing Miao (苗兵 University of Chinese Academy of Sciences)
15:40-16:00	Tea Break & Posters
Topic: Monte Carlo simulations/Network RG methods	
Chair: Jonathan Machta (University of Massachusetts Amherst, USA)	
16:00-16:30	<i>The worm process for the Ising model is rapidly mixing</i> Timothy Garoni (Monash University, Australia)
16:30-17:00	<i>How Can We Guide the Worm in a Lattice Model</i> Hidemaro Suwa (University of Tokyo, Japan)
17:00-17:30	<i>Neural Network Renormalization Group</i> Lei Wang (王磊 Institute of Physics, Chinese Academy of Sciences)
17:30-17:50	<i>Overlap of two topological phases in the antiferromagnetic Potts model</i> Cheng-Xiang Ding (丁成祥 Anhui University of Technology)
17:50-18:10	<i>Finite-size scaling above the upper critical dimension</i> Zong-Zheng Zhou (周宗政 Monash University, Australia)

6th April	
Topic: Novel methodologies	
Chair: Xiao-Song Chen (陈晓松)	
8:15-8:45	<i>Machine Learning of Quantum and Topological Physics</i> Hui Zhai (翟荟 Institute of Advanced Studies, Tsinghua University)
8:45-9:15	<i>Born Machine: Unsupervised Generative Modeling using Matrix Product States</i> Pan Zhang (张潘 Institute of Theoretical Physics, Chinese Academy of Sciences)
9:15-9:45	<i>BLUES function method in physics</i> Joseph Indekeu (Institute for Theoretical Physics, KU Leuven, Belgium)
9:45-10:15	<i>Simulating topological phase transition of lattice models in linear circuit networks</i> Jie Ren (任捷 Tongji University, China)
10:15-10:35	Tea Break
Topic: Monte Carlo/Tensor RG methods	
Chair: Peter Young (University of California, USA)	
10:35-11:05	<i>Population Annealing: A fast algorithm for spin systems with rough free energy landscapes</i> Jonathan Machta (University of Massachusetts Amherst, USA)
11:05-11:35	<i>Clock Monte Carlo Methods</i> Manon Michel (Ecole Normale Supérieure, France)
11:35-12:05	<i>Random numbers and large-scale Monte-Carlo</i> Lev Shchur(Landau Institute for Theoretical Physics, Russia)
12:05-12:25	<i>Nested Tensor Network Method and its applications</i> Zhi-Yuan Xie (谢志远 Renmin University of China)
12:25-14:00	Lunch Break
Topic: Network science	
Chair: Ye Wu (吴晔)	
14:00-14:30	<i>Explosive synchronization and Bellerophon states</i> Shu-Guang Guan (管曙光 East China Normal University)
14:30-15:00	<i>The Spreading Dynamics of Forget-Remember Mechanism</i> Wei Li (李炜 Central China Normal University)
15:00-15:30	<i>Universal state equation of quasi-particle gas for American domestic passenger flights</i> Chen-Ping Zhu (朱陈平 Nanjing University of Aeronautics and Astronautics)
15:30-15:50	<i>Scaling in complex network and explosive percolation</i> Ding-Ding Han (韩定定 East China Normal University)
15:50-16:10	<i>Phase transitions in cooperative percolations</i> Li Chen (陈理 Shaanxi Normal University)
16:10-16:30	<i>Improved recommendation algorithms based on data feature mining</i> Tian Qiu (邱天 Nanchang Aeronautical University, China)
16:30-16:50	Tea Break
Chair: Henk Bloete (Leiden University, Netherlands)	
16:50-17:20	<i>Critical and Griffiths-McCoy singularities in quantum Ising spin-glasses on d-dimensional hypercubic lattices: A series expansion study</i> Peter Young (University of California, USA)
Poster	
18:00-20:00	Banquet & 75th-birthday party of Henk Bloete

7th April	
Topic: Disordered systems	
Chair: Li-Mei Xu (徐莉梅 Peking University)	
8:15-8:45	<i>The Ising spin glass: new methods for old models and old methods for new models</i> Martin Weigel (Coventry University, UK)
8:45-9:15	<i>Quantum vs Classical Optimization</i> Helmut G. Katzgraber (Texas A&M University, USA)
9:15-9:45	<i>Percolation, Frustration, and Computational Complexity</i> Hai-Jun Zhou (周海军 Institute of Theoretical Physics, Chinese Academy of Sciences)
9:45-10:05	<i>The saddle point solution of the Landau-Ginzburg-Wilson for McCoy-Wu Ising model with disorder</i> Xin-Tian Wu (吴新天 Beijing Normal University)
10:05-10:25	Tea Break
Topic: Quantum systems	
Chair: Xi-Wen Guan (管习文 Wuhan Institute of Physics and Mathematics of Chinese Academy of Sciences)	
10:25-10:55	<i>Decoding quantum criticalities from fermionic/parafermionic topological states</i> Guang-Ming Zhang (张广铭 Tsinghua University)
10:55-11:25	<i>Quantum Monte Carlo study of Disordered Spin Systems</i> Dao-Xin Yao (姚道新 Sun Yat-sen University)
11:25-11:55	<i>Quantum phase transition in the spin-boson model</i> Qing-Hu Chen (陈庆虎 Zhejiang University)
11:55-12:15	<i>Universal Scaling and Critical Exponents of the Anisotropic Quantum Rabi Model</i> Mao-Xin Liu (刘卯鑫 Beijing Computational Science Research Center)
12:15-14:00	Lunch Break
Topic: Non-equilibrium systems	
Chair: Zhan-Chun Tu (涂展春 Beijing Normal University)	
14:00-14:30	<i>Landau-Lifshitz-Gilbert equation and phase transitions</i> Bo Zheng (郑波 Zhejiang University, China)
14:30-15:00	<i>Physics of Kuramoto oscillators</i> Yue-Heng Lan (兰岳恒 Beijing University of Posts and Telecommunications)
15:00-15:30	<i>Synchronic patterns in coupled systems</i> Xin-Gang Wang (王新刚 Shaanxi Normal University, China)
15:30-15:50	<i>Application of a variant lattice Boltzmann method to nonequilibrium reacting flows</i> Chuan-Dong Lin (林传栋 Tsinghua University, China)
15:50-16:10	Tea Break & Posters
Topic: Quantum Systems	
Chair: Wen-An Guo (郭文安 Beijing Normal University)	
16:10-16:30	<i>Quantum Spin Liquid with Even Ising Gauge Field Structure on Kagom Lattice</i> Yan-Cheng Wang (王艳成 China University of Mining and Technology)
16:30-16:50	<i>Dynamic Localization in Spin Clusters</i> Kai Ji (吉凯 Shanghai Normal University, China)
16:50-17:20	<i>Probing spin charge separation with quantum criticality</i> Xi-Wen Guan (管习文 Wuhan Institute of Physics and Mathematics of Chinese Academy of Sciences)
17:20-17:50	<i>Critical behavior in a model of free parafermions</i> Murray Batchelor (Chongqing University, China)
17:50-18:00	Closing
	Youjin Deng (邓友金)

Poster session

1. Logarithmic Rainbow Free Energy on a Topological Manifold
Hao-Xin Wang (王昊昕)
2. Conformal Thermal Tensor Network and Universal Entropy on Topological Manifolds
Lei Chen (陈磊)
3. The study of phase transitions in the Equivalent-neighbor 4-state Potts models in two dimensions
Yu-Hai Liu (刘玉海)
4. Universal entropy of conformal critical theories on a Klein bottle: a quantum Monte Carlo study
Wei Tang (唐维)
5. Quantum quenching kinetics of quantum entanglement on a spin chain with Dzyaloshinskii-Molriya interaction
Yu-Liang Xu (徐玉良)
6. Medium-range Potts models in two dimensions
Xiao-Feng Qian (钱小凤)
7. Active image restoration
Rongrong Xie (谢容容), Shengfeng Deng (邓盛锋), Weibing Deng (邓为炳) and Armen E.
8. The non-equilibrium universality class study of forget-remember mechanism on the (1+1)-d lattice
Jianming Shen (申建民), Shengfeng Deng (邓盛锋), Wei Li (李炜)

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Critical behavior in a model of free parafermions

Murray Batchelor

Chongqing University and Australian National University

In 1989 Rodney Baxter discovered a simple N -state quantum spin chain with eigenvalue spectrum given by a direct product of $N \times N$ diagonal matrices. It was not until 2014 that Paul Fendley showed that this structure corresponds to previously elusive free parafermions. In this talk I will discuss the critical properties of this remarkable model. For $N = 2$ the model reduces to the familiar quantum Ising chain in a transverse field described by free fermions. For $N > 2$ the model is non-hermitian and displays boundary-dependent critical behavior.

[1] R. J. Baxter, Phys. Lett. A 140, 155 (1989)

[2] P. Fendley, J. Phys. A 47, 075001 (2014)

[3] F. C. Alcaraz, M. T. Batchelor and Z.-Z. Liu, J. Phys. A 50, 16LT03 (2017)

[4] F. C. Alcaraz and M. T. Batchelor, Anomalous bulk behavior in the free parafermion $Z(N)$ spin chain, arXiv:1802.04453

The zoo of two-dimensional $O(n)$ models

Henk W. J. Blöte

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The two-dimensional $O(n)$ model displays a considerable variety of universality classes. These may depend on the introduction of a different form of $O(n)$ -symmetric couplings, and on the mapping of the spin model on the $O(n)$ loop model. Even after several decades of research efforts on relatively simple $O(n)$ lattice models, it is still possible to discover new phenomena, depending on symmetry and topological properties rooted in the microscopic structure of the pertinent model.

PACS numbers: 64.60.Cn, 64.60.De, 64.60.F-, 75.10.Hk

The halon: a quasiparticle featuring critical charge fractionalization

Kun Chen

The halon is a special critical state of an impurity in a quantum-critical environment. The hallmark of the halon physics is that a well-defined integer charge gets fractionalized into two parts: a microscopic core with half-integer charge and a critically large halo carrying a complementary charge of $\pm 1/2$. The halon phenomenon emerges when the impurity--environment interaction is fine-tuned to the vicinity of a boundary quantum critical point (BQCP), at which the energies of two quasiparticle states with adjacent integer charges approach each other. The universality class of such BQCP is captured by a model of pseudo-spin- $1/2$ impurity coupled to the quantum-critical environment, in such a way that the rotational symmetry in the pseudo-spin xy -plane is respected, with a small local "magnetic" field along the pseudo-spin z -axis playing the role of control parameter driving the system away from the BQCP. On the approach to BQCP, the half-integer projection of the pseudo-spin on its z -axis gets delocalized into a halo of critically divergent radius, capturing the essence of the phenomenon of charge fractionalization. With large-scale Monte Carlo simulations, we confirm the existence of halons---and quantify their universal features---in $O(2)$ and $O(3)$ quantum critical systems.

Conformal Thermal Tensor Network and Universal Entropy on Topological Manifolds

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Thermal quantum critical systems, with partition functions expressed as conformal tensor networks, are revealed to exhibit universal entropy corrections on nonorientable manifolds. Through high-precision tensor network simulations of several quantum chains, we identify the universal entropy $S_K = \ln k$ on the Klein bottle, where k relates to quantum dimensions of the primary fields in conformal field theory (CFT). Different from the celebrated Affleck-Ludwig boundary entropy $\ln g$ (g reflects noninteger ground-state degeneracy), S_K has no boundary dependence or surface energy terms accompanying it, and can be very conveniently extracted from thermal data. On the Möbius-strip manifold, we uncover an entropy $S_M = \frac{1}{2}(\ln g + \ln k)$ in CFT, where $\frac{1}{2}\ln g$ is associated with the only open edge of the Möbius strip and $\frac{1}{2}\ln k$ is associated with the nonorientable topology. As a useful application, we employ the universal entropy to accurately pinpoint the quantum phase transitions, even for those without local order parameters.

Phase transitions in cooperative percolations

Li Chen

Shaanxi Normal University

Contagion processes, such as the spread of infectious diseases, exhibit a percolation transition, which separates transitory prevalence from outbreaks that reach a finite fraction of the population. Such transitions are commonly believed to be continuous, but empirical evidences have shown more violent modes when the participating diseases are not limited to one kind, such as the co-infection between the Spanish flu and pneumonia, the concurrent circulation of HIV and a host of other diseases. Here we propose a percolation model involving two different agents and inter-agent cooperation. Our simulations revealed the possible emergence of a massive avalanche-like outbreak right at the threshold, manifested as a discontinuous percolation transition. The underlying structures are found to be influential to the nature of phases transitions; a regular lattice of 2d can only exhibit continuous transitions, while a lattice of higher dimension could be discontinuous. Other disordered structures, such as Erdős-Reny networks, will also be briefly discussed. A mean field theory is also provided. These findings based on cooperative percolations are of practice relevance in many contagion processes in the real world.

Related publications:

- [1] L Chen, F Ghanbarnejad, W Cai, P Grassberger, Outbreak of coinfections: the critical role of cooperativity, *Europhys. Lett.* 104, 50001 (2013).
- [2] W Cai, L Chen, F Ghanbarnejad, P Grassberger, Avalanche outbreaks emerging in cooperat

Quantum phase transitions of the spin-boson model

Qing-Hu Chen

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The celebrated spin-boson model describes a qubit coupled with a dissipative environment represented by a continuous bath of bosonic modes and undergoes continuous quantum phase transitions in the sub-Ohmic bath regime. But the criticality is still highly controversial to date[1]. The genuine description of the ground state is still lacking at least in the analytical sense. Several advanced numerical approaches have been developed [2-5].

In this talk, I will talk about our recent new approach where the well-known Silbey-Harris (SH) polaron ansatz for the spin-boson model is improved by adding orthogonal displaced Fock states [6]. The obtained results for the ground-state in all baths converge very quickly within finite displaced Fock states and corresponding SH results are corrected considerably. Especially for the sub-Ohmic spin-boson model, the converging results are obtained for $0 < s < 1/2$ in the fourth-order correction and very accurate critical coupling strengths of the quantum phase transition are achieved. Since the present improved SH ansatz can yield very accurate, even almost exact results, it should have wide applications and extensions in various spin-boson model and related fields.

- [1] S. Kirchner, K. Ingersent, and Q. Si, Phys. Rev. B 85, 075113 (2012)
- [2] R. Bulla, N.-H. Tong, and M. Vojta, Phys. Rev. Lett. 91, 170601 (2003).
- [3] A. Winter, H. Rieger, M. Vojta, and R. Bulla, Phys. Rev. Lett. 102, 030601 (2009).
- [4] A. Alvermann and H. Fehske, Phys. Rev. Lett. 102, 150601 (2009)
- [5] Y.-Y. Zhang, Q.-H. Chen, and K.-L. Wang, Phys. Rev. B 81, 121105(R) (2010).
- [6] Shu He, Liwei Duan, Qing-Hu Chen; Phys. Rev. B 97, 115157 (2018)

Finite-Size Scaling of Correlation Function

Xiao-Song Chen

For a fine system near its critical point, we propose a finite-scaling form of correlation function. In the Ising model and the bond percolation in two-dimensional lattices, this finite-size scaling form has been confirmed by our Monte Carlo simulations. From the ratio of correlation functions in a system with different system sizes, the critical point of this system can be determined by the fixed point of ratio.

Overlap of two topological phases in the antiferromagnetic Potts model

Ran Zhao¹, Chengxiang Ding^{2*}, and Youjin Deng¹

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By controlling the vortex core energy, the three-state ferromagnetic Potts model can exhibit two types of topological paradigms, including the quasi-long-range ordered phase and the vortex lattice phase [PRL 116, 097206 (2016)]. Here, by Monte Carlo simulations using an effective worm algorithm, we show that by controlling the vortex core energy, the antiferromagnetic Potts model can also exhibit the two topological phases, more interestingly, the two topological phases can overlap with each other.

The worm process for the Ising model is rapidly mixing

A. Collecchio, Timothy Garoni, T. Hyndman and D. Tokarev

We consider the worm process for the zero-field ferromagnetic Ising model, introduced by Prokofiev and Svistunov. We prove the process is rapidly mixing on all finite graphs and at all temperatures. As a corollary, we construct fully-polynomial randomized approximation schemes for the Ising susceptibility and two-point correlation function.

Explosive synchronization and Bellerophon states

Shu-Guang Guan

East China Normal University

耦合振子系统通过同步可以形成宏观有序的协同态。过去对协同态的研究大多数都集中于平稳态 (stationary state)，如同步态、行波态等，而对非平稳 (non-stationary state)、时变的高阶协同态研究不足。2016 年，华师大小组报道了一种新的奇异协同态——Bellerophon 态，它具有三个基本性质：分离性、时变性和异质性。在这次会议上我将重点报道这一工作，以及关于 Bellerophon 态研究的一些最新进展。

Probing spin charge separation with quantum criticality

X. W. Guan (管习文)

*Wuhan Institute of Physics and Mathematics, Chinese Academy of Science;
Department of Theoretical Physics, Research School of Physics and Engineering,
Australian National University, Canberra*

The spin charge separation is the hall mark of the low energy physics in one dimension (1D). However, such a unique 1D phenomenon still lacks a comprehensive understanding in experiment. In this talk, we will discuss this novel quantum separation with quantum criticality of ultracold atoms. We will show that two-component ultracold Fermi and Bose gases exhibit different spin charge separation mechanisms. The former can form two Luttinger liquids (LLs) with different propagation velocities in charge and spin degrees of freedom. Whereas, the latter gives rise to the separation of a single LL from the bound states of spin waves. Such difference result in significantly different quantum power law scalings in magnetic and thermal properties, providing a fundamental understanding of quantum criticality and quantum liquids.

Scaling in the vicinity of the four-state Potts fixed point

WenAn Guo (郭文安)

Beijing Normal University

The pure critical Baxter-Wu model displays the critical behavior of the four-state Potts fixed point in two dimensions without logarithmic corrections. The introduction of different couplings in the up- and down triangles moves the model away from this fixed point, so that logarithmic corrections appear. Real couplings move the model into the first-order range. In this talk, we will show that, by introducing complex couplings, the model is brought in the opposite direction characterized by the same type of logarithmic corrections as present in the four-state Potts model. Our finite-size analysis basing on transfer matrix calculations confirms in detail the existing renormalization theory describing the immediate vicinity of the four-state Potts fixed point.

Entropy Stabilizes Floppy Crystals of Mobile DNA-Coated Colloids

Hao Hu, Pablo Sampedro Ruiz, and Ran Ni

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Grafting linkers with open ends of complementary single-stranded DNA makes a flexible tool to tune interactions between colloids, which facilitates the design of complex self-assembly structures. Recently, it has been proposed to coat colloids with mobile DNA linkers, which alleviates kinetic barriers without high-density grafting, and also allows the design of valency without patches. However, the self-assembly mechanism of this novel system is poorly understood. Using a combination of theory and simulation, we obtain phase diagrams for the system in both two and three dimensional spaces, and find stable floppy square and CsCl crystals when the binding strength is strong, even in the infinite binding strength limit. We demonstrate that these floppy phases are stabilized by vibrational entropy, and “floppy” modes play an important role in stabilizing the floppy phases for the infinite binding strength limit. This special entropic effect in the self-assembly of mobile DNA-coated colloids is very different from conventional molecular self-assembly, and it offers a new axis to help design novel functional materials using mobile DNA-coated colloids.

Keywords: self-assembly, colloidal crystal, grafted colloids

Reference: H. Hu, P. Sampedro Ruiz, R. Ni, *Phys. Rev. Lett.* **120**, 048003 (2018).

BLUES function method in physics

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We introduce a computational method in physics that goes ‘beyond linear use of equation superposition’ (BLUES). A BLUES function is defined as a solution of a nonlinear differential equation (DE) with a delta source that is at the same time a Green’s function for a related linear DE. For an arbitrary source, the BLUES function can be used to construct an exact solution to the nonlinear DE with a different, but related source. Alternatively, the BLUES function can be used to construct an approximate piecewise analytical solution to the nonlinear DE with an arbitrary source. For this alternative use the related linear DE need not be known. The method is illustrated in a few examples using analytical calculations and numerical computations. Areas for further applications are suggested.

Keywords: nonlinear differential equation, Green’s function, sources

Dynamic Localization in Spin Clusters

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Dynamics of many-particle quantum systems under periodic perturbations is a fascinating subject that has recently attracted renewed attention in various contexts. An important task in this aspect is to identify novel and peculiar quantum behaviors under periodic driving. A relevant phenomenon here is dynamic localization (DL), which is the time-domain analog of Anderson localization. DL is already well understood for systems with one or a few degrees of freedom, such as kicked quantum rotator. While, DL in many-particle systems is still an open issue up to date. If it occurs, one would expect that the system stops absorbing energy from external driving. The aim of current work is to understand the applicability of DL to the many-particle systems. We have investigated numerically and analytically the heating processes in the interacting spins 1/2 clusters subjected to periodic pulses of external magnetic field. We found that the heating caused by the periodic kicks with long delays was asymptotically much slower than that caused by slightly aperiodic kicks. We attributed the above difference to DL. Moreover, our findings indicate that there is a threshold for the pulse strength below which the heating can be suppressed. This threshold decreases with the increase of the cluster size, approaching zero in the thermodynamic limit. We obtain the above threshold analytically as a condition for the breakdown of the golden rule in the second-order perturbation theory [1].

Key words: Dynamic localization, Floquet theorem, Spin cluster

Reference

K. Ji and B. V. Fine, arXiv:1712.10028.

Quantum vs Classical Optimization

Helmut G. Katzgraber
Texas A&M University, USA

Can quantum computers meet the tantalizing promise of solving complex calculations - such as optimization problems - faster than classical computers based on transistor technologies? Although we are closer to answering this question within the next few years, medium-scale useful quantum devices are still a distant reality. Inspired by these quantum developments, a variety of classical optimization techniques have emerged recently. In this talk an overview of the synergy between classical and quantum optimization is given with an emphasis on the study of spin-glass-like optimization problems.

Physics of Kuramoto oscillators

Yue-Heng Lan (兰岳恒)

Beijing University of Posts and Telecommunications

The mean-field theory describes well the synchronization of the Kuramoto oscillator model but is hard to accommodate fluctuations of the finite-sized system. We apply a special form of the RG theory recently developed for the differential dynamical systems to the finite size scaling observed numerically in the Kuramoto model and obtained interesting insights.

The Spreading Dynamics of Forget-Remember Mechanism

S. F. Deng and W. Li (李炜)

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We study extensively the forget-remember mechanism (FRM) for message spreading, originally introduced in [Eur. Phys. J. B **62**, 247 \(2008\)](#). The freedom of specifying forget-remember functions governing the FRM can enrich the spreading dynamics to a very large extent. The master equation is derived for describing the FRM dynamics. By applying the mean field techniques, we have shown how the steady states can be reached under certain conditions, which agrees well with the Monte Carlo simulations. The distributions of forget and remember times can be explicitly given when the forget-remember functions take linear or exponential forms, which might shed some light on understanding the temporal nature of diseases like flu. For time-dependent FRM there is an epidemic threshold related to the FRM parameters. We have proven that the mean field critical transmissibility for the SIS model and the critical transmissibility for the SIR model are the lower and the the upper bounds of the critical transmissibility for the FRM model, respectively.

Keywords: spreading dynamics; percolation; phase transition

Reference: PHYSICAL REVIEW E **95**, 042306 (2017)

Application of a variant lattice Boltzmann method to nonequilibrium reacting flows

Chuandong Lin

Reacting flows are ubiquitous in energy and environment systems where both hydrodynamic and thermodynamic nonequilibrium effects usually play essential roles from micro-, to meso-, and to macroscopic levels. Traditional numerical methods for reacting flows refer to the Navier-Stokes (NS) model based upon the continuum hypothesis, but they often meet with physical inaccuracy due to the significant nonequilibrium behaviors. As a kinetic methodology, a variant lattice Boltzmann model (VLBM) presents an accurate and efficient tool to simulate nonequilibrium reacting flows. Different traditional lattice Boltzmann method, which is widely used as a solver of various partial differential equations, the recently proposed VLBM could not only recover the macroscopic governing equations (such as the NS equations), but also capture more detailed thermodynamic nonequilibrium effects beyond the macroscopic governing equations. Hence, the application of the promising VLBM to nonequilibrium reacting flows has both academic meanings and practical values.

Universal Scaling and Critical Exponents of the Anisotropic Quantum Rabi Model

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We investigate the quantum phase transition of the anisotropic quantum Rabi model, in which the rotating and counterrotating terms are allowed to have different coupling strengths. The model interpolates between two known limits with distinct universal properties. Through a combination of analytic and numerical approaches, we extract the phase diagram, scaling functions, and critical exponents, which determine the universality class at finite anisotropy (identical to the isotropic limit). We also reveal other interesting features, including a superradiance-induced freezing of the effective mass and discontinuous scaling functions in the Jaynes-Cummings limit. Our findings are extended to the few-body quantum phase transitions with $N > 1$ spins, where we expose the same effective parameters, scaling properties, and phase diagram. Thus, a stronger form of universality is established, valid from $N = 1$ up to the thermodynamic limit.

The study of phase transitions in the Equivalent-neighbor 4-state Potts models in two dimensions

Yuhai Liu , Wenan Guo

Beijing Normal University

Henk W. J. Blöte

Lorentz Institute, Leiden University

We study the phenomenon that the universal behaviors of phase transitions vary with a variable interaction range in the Equivalent-neighbor 4-state Potts models on the square lattice . We locate the transition points and calculate the critical exponents for several interaction ranges as expressed by the number z of equivalent neighbors using Monte Carlo numerical simulations and finite-size scaling . Then we classify these phase transitions into two different classes . We find for $z \leq 20$, the phase transitions fit well in the short-range 4-state Potts model universality class ; For $z \geq 24$, the phase transitions become discontinuous and we find mean-field-like critical behavior .

From Critical Phenomena to Colloids: Phase separation out of equilibrium

Erik Luijten

Northwestern University

Colloidal suspensions are a prototypical example of systems that can be either passive or active. Here, I will demonstrate how various forms of dynamics and different types of interactions result in unexpected and until now largely unexplored aggregation and phase behavior. These observations, obtained through a combination of experiments and computer simulations, reveal striking connections between colloidal self-assembly and collective dynamics, and between dynamic behavior, classical thermodynamics, and critical phenomena. Moreover, a remarkable variety of collective dynamics can be realized through simple variation of the applied electric fields. The findings presented here provoke new thoughts on the nature of “soft” materials and our ability to manipulate them, and on the concept of “temperature” in non-equilibrium systems.

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Behavior of the specific heat in the Ising model for $d \geq 4$

Per-Håkan Lundow
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For $d \geq 4$, the critical exponents take their mean field values and thus $\alpha = 0$. It is rigorously known that for $d \geq 5$ the specific heat is also bounded. In fact, it shows a jump-discontinuity at T_c with different left- and right-hand limits, as in the mean-field model, and singular exponents describe how the limits are approached. We have estimated the limits and the exponents from simulation data for $d=5,6,7$ along with estimates of the critical temperature. For $d=4$ an unbounded specific heat with a logarithmic singularity at T_c has been predicted but not rigorously proven. Some of our canonical ensemble data are consistent with this prediction but microcanonical ensemble data lend support to a bounded specific heat.

Duality and the universality class of the three-state Potts antiferromagnet on plane quadrangulations

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We provide a new criterion based on graph duality to predict whether the 3-state Potts antiferromagnet on a plane quadrangulation has a zero- or finite-temperature critical point, and its universality class^[1]. The former case occurs for quadrangulations of self-dual type, and the zero-temperature critical point has central charge $c=1$. The latter case occurs for quadrangulations of non-self-dual type, and the critical point belongs to the universality class of the 3-state Potts ferromagnet. We have tested this criterion against high-precision computations on four lattices of each type, with very good agreement. We have also found that the Wang-Swendsen-Kotecky algorithm has no critical slowing-down in the former case, and critical slowing-down in the latter. The details will be reported in Ref.[2].

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[2] Jian-Ping Lv, Youjin Deng, Jesper Lykke Jacobsen, Jesús Salas, in preparation

Population Annealing: A fast algorithm for spin systems with rough free energy landscapes

Jonathan Machta

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Population annealing is a sequential Monte Carlo algorithm for equilibrium sampling. Population annealing is a parallel version of simulated annealing in which a large population of replicas is simultaneously taken through an annealing schedule. Resampling (i.e. differential reproduction of replicas) is carried out at each annealing step and ensures that the population of replicas remains close to an equilibrium (e.g. canonical or microcanonical) ensemble. Population annealing is a highly efficient method for simulations of systems with rough free energy landscapes. In this talk I will describe the algorithm and its properties, discuss its convergence to equilibrium, and present results obtained for Ising spin glasses.

Factorizing transitions in MCMC, the key to irreversibility and scalability

Manon Michel

Ecole Normale Supérieure, France

Recently developed, irreversible Monte Carlo schemes are now under a growing attention. One of the ground concepts is the factorization of the transitions probabilities, which allows to extract information and symmetries from what can be seen now as the black box of Metropolis transition probabilities. Today, the advantage of factorizing transitions appears now more general than their implementation in irreversible schemes, as the factorization has been shown to be the general key to the reduction of the computational complexity of irreversible continuous-time as well as reversible discrete-time schemes.

Phase transitions in a confined polyelectrolyte solution

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Polyelectrolyte solution is a model system to study phase behaviors of biosystems in statistical physics. Due to the long-range Coulomb interaction, a polyelectrolyte solution can undergo a local phase separation to create ordered structures in nanoscale. On the other hand, charge screening destroys the local order and leads the solution to global phase separation. This defines the Lifshitz behavior in the particular system, wherein both the Ising and the Brazovskii universality class are admitted. We discuss phase transitions in this system within the Landau phase transition theory and, in particular, finite size effects are studied by confining the system in a nanoscale space.

Key words: phase transition, critical phenomena, finite size effect, Coulomb system

Improved recommendation algorithms based on data feature mining

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An efficient way to improve personalized recommendation is to properly incorporate the data feature into algorithm design. In our study, we investigate the user interest pattern based on four recommender system data. A strong preference of active users to cold objects and a wide interest pattern of inactive users are revealed. Based on the user interest pattern, a general formula is proposed to improve seven recommendation algorithms with different similarity measures. Nearly all the new algorithms are found to outperform their previous ones in recommendation accuracy, diversity, and accuracy of the cold objects, except for the diversity compared with a heat conduction algorithm. Moreover, one of the improved algorithms is also more advantageous than two other excellent algorithms.

Keyword: Infophysics; Recommender system; Big data

Simulating topological phase transition of lattice models in linear circuit networks

Jie Ren (任捷)

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Quantum spin Hall effect (QSHE) of electrons has improved the development of condensed matter research nowadays, which describe spin-dependent quantum transport behavior in solid state. Based on metamaterials, the essential physical mechanism behind QSHE of classical waves can be understood deeply and verified easily. I will present, for example, the topological interface mode between two topologically-distinct phase, i.e., permittivity negative and permeability negative metamaterials, and the topological phase transition and QSHE in topological circuits. The latter includes the topological Lieb lattice, a line-centered square lattice with rich topological properties, in a radio-frequency circuit. We design a specific capacitor-inductor connection to resemble the intrinsic spin-orbit coupling and construct the analog spin by mixing degrees of freedom of voltages. As such, one is able to simulate the QSHE in the topological Lieb lattice of linear circuits. We then investigate the spin-resolved topological edge mode and the topological phase transition of the band structure varied with capacitances. Finally, we discuss the extension of the $\pi/2$ phase change of hopping between sites to arbitrary phase values. Our results may find implications in engineering microwave topological metamaterials for signal transmission and energy harvesting.

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Random numbers and large-scale Monte-Carlo

Lev Shchur

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Monte Carlo simulations needs random numbers. History shows how problems with the quality of the pseudo-random numbers (PRN) biased results of Monte Carlo simulations. It happens from time to time, and connected with the enlargements of the scale of simulations. Many improvements has been done to make random number generators (RNG) more reliable and safe. At the moment we have a good pile of the good random number generators, and huge battery of tests in the libraries. Emerging computing systems are extremely parallel. The total computing power is still following to the Moore law thanks to the multi-core/multi-thread and hybrid nature of computers. Attention should be paid for the possible influence of the correlations due to the high parallelism in the simulations. We still do not have reasonable algorithms and tests based on them to qualify RNGs for the parallel simulations. Not much attention paid to the possible correlations between parallel streams of PRNs. In the presentation we will present discussion of the problem. We present solutions for the massive parallel simulations. We present details of our approach [1] and the corresponding libraries [2,3]. Examples will include generation of uncorrelated parallel streams of PRNs using CPU, GPU, and CPU, its extensions SSE2, AVX2 [4], and new AVX512 technology. Explanation of our interest to the problem of RNGs coming back to the time of our collaboration with Henk Blöte in 90-s, when we build special purpose computers for the 3d Ising model [5]. The success in the understanding of the nature of correlations [6,7] gives us possibility to get the level precision in the estimation of critical exponents, temperature, and observables of 3D Ising model [8], which still is not bitten with the modern computing power.

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Conformal Bootstrap Approach to Critical Phenomena

Ning Su

Conformal field theories (CFT) describe a large class of second order phase transitions. In last few years, based on the idea of bootstrap, a powerful non-perturbative numerical method have been developed to attack CFTs. In this talk, I will explain basic logic of bootstrap and show it constrains CFT data. I then discuss possible implications for phase

How Can We Guide the Worm in a Lattice Model

Hidemaro Suwa

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(Department of Physics and Astronomy, The University of Tennessee)

The worm algorithm is a versatile technique in use of the Markov chain Monte Carlo method for both quantum and classical systems. In particular, the dynamic critical exponents of classical spin systems are greatly reduced, compared to the case of a single spin update. We have improved the efficiency of the worm algorithm for classical models in combination with the directed-loop framework and the geometric probability optimization. Performance improvement is demonstrated for the Ising model at the critical temperature by measurement of exponential autocorrelation times and asymptotic variances; the well-guided worm is approximately 25 times as efficient as the classical worm for the simple-cubic-lattice model. Remarkably, it is even more efficient than the Wolff cluster algorithm. We have estimated the dynamic critical exponent of the longest time scale in the worm update to be $z \sim 0.27$.

In this talk, I will introduce the geometric probability optimization[1], which can be applied to any Markov chain Monte Carlo update and provide optimized probabilities even without detailed balance. Our directed worm[2] is applicable to a wide range of physical systems, such as the ϕ^4 model, the Potts model, the $O(n)$ loop model, the lattice QCD, and also some frustrated systems in combination with the dual worm formalism.

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**Universal entropy of conformal critical theories on a
Klein bottle: a quantum Monte Carlo study**

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Recently, entropy corrections on nonorientable manifolds such as the Klein bottle are proposed as a universal characterization of critical systems with an emergent conformal field theory (CFT). We show that entropy corrections on the Klein bottle can be interpreted as a boundary effect via transforming the Klein bottle into an orientable manifold with nonlocal boundary interactions, based on which we propose an efficient quantum Monte Carlo (QMC) method to extract such universal boundary entropies. For rational CFTs, we perform the QMC simulation in the quantum q -state Potts model with $q=2,3,4$ and compare the calculation results with the CFT predictions. Beyond the rational CFT, we prove that in the compactified boson CFT, when the compactification radius is not a rational number, the universal Klein bottle entropy still exists, and it depends only on the compactification radius. To verify this CFT prediction, we perform QMC simulations in the XXZ model and compare the numerical results with the CFT predictions. Due to the direct connection between the compactification radius and the Luttinger parameter, the relation between the compactification radius and the Klein bottle entropy also provides a new method to extract the Luttinger parameter from lattice models.

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Logarithmic Rainbow Free Energy on a Topological Manifold

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(Dated: February 1, 2018)

Neural Network Renormalization Group

Lei Wang

I will present a variational renormalization group approach using deep generative model composed of bijectors. The model can learn hierarchical transformations between physical variables and renormalized collective variables. It can directly generate statistically independent physical configurations by iterative refinement at various length scales. The generative model has an exact and tractable likelihood, which provides renormalized energy function of the collective variables and supports unbiased rejection sampling of the physical variables. To train the neural network, we employ probability density distillation, in which the training loss is a variational upper bound of the physical free energy. The approach could be useful for automatically identifying collective variables and effective field theories.

Synchronous patterns in complex networks

Xingang Wang

School of Physics and Information Technology,

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In the synchronization of coupled nonlinear oscillators, a general finding is that as the oscillators become non-identical, the propensity for synchronization will be deteriorated. Yet, parameter mismatch is unavoidable in realistic complex systems and, in some cases, the oscillators are even of different types of dynamics. The contradiction between theory and reality poses the following paradox for oscillator synchronization: how non-identical oscillators are synchronized in a complex network? Here, by the model of coupled chaotic oscillators, we are able to demonstrate numerically and argue analytically that, despite the parameter mismatch (different dynamics) of the oscillators, stable synchronous patterns can still be generated in complex networks. A general framework is proposed for the identification and stability analysis of synchronous patterns in the general complex networks, with the theoretical predications in good agreement with the numerical results. The study sheds lights on the collective behaviors of coupled non-identical oscillators, and might have implications the functionality and operation of many realistic systems, e.g., the power-grid and the human brain network.

Quantum Spin Liquid with Even Ising Gauge Field Structure on Kagome Lattice

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Employing large-scale quantum Monte Carlo simulation, we study the extended XXZ model on the kagome lattice. A Z_2 quantum spin liquid phase with effective even Ising gauge field structure emerges from the delicate balance among three symmetry breaking phases including stripe solid, staggered solid and ferromagnet. This Z_2 spin liquid is stabilized by an extended interaction related to the Rokhsar-Kivelson potential in the quantum dimer model limit. The phase transitions from the staggered solid to a spin liquid or ferromagnet are found to be first-order, so is the transition between the stripe solid and ferromagnet. However, the transition between spin liquid and ferromagnet is found to be continuous and belongs to the 3D XY * universality class associated with the condensation of spinons. The transition between spin liquid and stripe solid appears to be continuous and associated with the condensation of visons.

The Ising spin glass: new methods for old models and old methods for new models

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The Ising spin glass in 2D exhibits rich behavior with subtle differences in the scaling for different coupling distributions. We use combinatorial optimization methods to determine exact ground states for systems with up to 10000×10000 spins. A combination of new algorithms allow us to treat samples with fully periodic boundaries and to sample uniformly from degenerate ground states for the $\pm J$ model. To establish a unified framework for studying both discrete and continuous coupling distributions in arbitrary dimensions, we introduce the binomial spin glass. In this model, the couplings are the sum of m identically distributed Bernoulli random variables. In the continuum limit $m \rightarrow \infty$, this system reduces to the Edwards-Anderson model with Gaussian couplings, while $m = 1$ corresponds to the $\pm J$ spin glass. Using this model, we derive a rigorous bound for the degeneracy of any energy level. Studying the defect energies in this model, we uncover intriguing subtleties in the behavior of the model with respect to the order in which the thermodynamic ($N \rightarrow \infty$) and continuum ($m \rightarrow \infty$) limits are taken.

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The saddle point solution of the Landau-Ginzburg-Wilson Hamiltonian for McCoy-Wu Ising model with disorder

Xin-Tian Wu

Department of Physics, Beijing Normal University

The two-dimensional Landau-Ginzburg-Wilson Hamiltonian with a random temperature only depending on one of the coordinates (x) is studied. It is the effective Hamiltonian of McCoy-Wu strip-random Ising model. The discrete difference saddle point equation is solved numerically. For a region with negative reduced temperature, the saddle point solution may be nonzero. These regions are locally ordered regions (LOR) and they are strip-like. Due to the discreteness there is a free energy barrier in the y -direction. This barrier may stabilize the excited solutions with domain wall cutting the LOR into pieces. The stability condition for such domain wall is studied. As temperature varies, the domain wall across a single LOR may become unstable. Then neighbored locally ordered regions may fuse into an elementary cluster, across which the domain wall is stable. The criterion for multiple LORs forming an elementary cluster is investigated. At the saddle point level, the effective Hamiltonian for the excited state solutions is a Ising model in which each elementary cluster is a spin chain and clusters interact weakly in x -direction. For random temperatures distributed uniformly with $w=0.5, 0.4, 0.3$, the distribution of cluster size, the couplings in x -direction and y -direction are studied numerically. For such random temperature, the clusters interact with each other like an Ising model with random bonds in x -direction and homogeneous bonds in y -direction. The y -direction bonds are extremely strong and those in x -direction are extremely weak. The distribution of y -direction couplings is exponential. The Griffiths-McCoy singularity can be explained in this framework.

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Nested Tensor Network Method and its applications

Zhi-Yuan Xie

In the tensor-network framework, the expectation values of two-dimensional quantum states or three-dimensional classical model are evaluated by contracting a double-layer tensor network constructed from initial and final tensor-network states. The computational cost of carrying out this contraction is generally very high, which limits the largest bond dimension of tensor-network states that can be accurately studied to a relatively small value. We propose Nested Tensor Network Method to solve this problem by mapping the double-layer tensor network onto an intersected single-layer tensor network. This reduces greatly the bond dimensions of local tensors to be contracted and improves dramatically the efficiency and accuracy of the evaluation of expectation values of tensor-network states. It almost doubles the largest bond dimension of tensor-network states whose physical properties can be efficiently and reliably calculated, and it extends significantly the application scope of tensor-network methods. We demonstrate its performance by the application in the spin-1/2 Kagome anti-ferromagnet system and the Ising model at simple cubic lattice.

Quantum quenching kinetics of quantum entanglement on a spin chain with Dzyaloshinskii-Molriya interaction

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By using the concept of concurrence and quantum renormalization group method, the time -evolution of quantum entanglement is investigated near the quantum critical point of Ising chain with Dzyaloshinskii-Molriya (DM) interaction. After importing two quantum quench, i.e., the sudden introduction of DM interaction or the rotation 180 °all spin along the X axis, the entanglement between the non-nearest-neighbor spin(or spin block) is calculated. It is found that there are some similar behaviors in the entanglement of different quantum quench modes: the entanglements periodically fluctuates with time under certain DM action. The effect of DM has an important influence on the dynamic characteristic of entanglement, i.e., the time of the entanglement evolution reaching the first maximum value. When the system size becomes large and especially the thermodynamic limit is approaching, the characteristic time has a mutation behavior at the quantum critical point, and its first derivative also exhibits strange behavior and finite-size scaling behavior. It is found that the scaling behavior of the time-evolution characteristic of the entanglement is similar to that of entanglement in the equilibrium state. After a certain quench time, the entanglement emerges a concussion increase with the increase of the DM interaction, and the concussion behaviors of the two different quenching modes are different. When the system becomes large, the concussion change of entanglement also can reflect that the quantum phase transition from saturated chiral order phase to antiferromagnetic Neel one occurs on the system. This work is supported by the National Natural Science Foundation of China under Grants No. 11675090 and the post-doctoral innovation foundation of Shandong Province.

Keywords: entanglement dynamics, quantum phase transition, Dzyaloshinskii-Molriya action, quantum renormalization group

Quantum Monte Carlo study of Disordered Spin Systems

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We use quantum Monte Carlo method to study the disordered $S=1/2$ quantum spins on the 1D and 2D lattices. For the 1D case, we study the random Heisenberg chains with standard Heisenberg couplings and multi-spin couplings. The QMC simulations demonstrate logarithmic corrections to the power-law decaying correlations obtained with the SDRG scheme. In the model with multispin couplings, where the clean system dimerizes spontaneously, random singlets form between spinons localized at domain walls in the presence of disorder. This amorphous valence-bond solid is asymptotically a random-singlet state. The dynamical properties of the random Heisenberg chain is studied by using the stochastic analytic continuation of quantum Monte Carlo results in imaginary time. We investigate the dynamic spin structure factor $S(q, \omega)$, which can be probed by inelastic neutron scattering and NMR experiments. For the 2D case, we study the disordered $S=1/2$ quantum spins on the square lattice with three different nearest neighbor interactions J_1 , J_2 and J_3 . Here J_1 represents weak bonds, and J_2 and J_3 correspond to stronger bonds which are randomly distributed on columnar rungs forming coupled 2-leg ladders. By tuning the average value of J_2 and J_3 , the system undergoes Neel-glass-paramagnetic quantum phase transition. A wide range of Mott glass phase has been found. We show that its uniform susceptibility in the glass phase follows $\chi \sim \exp(-b/T^\alpha)$, where $0 < \alpha < 1$. This dimerized disordered quantum spin system shows the violation of Harris criterion.

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Critical and Griffiths-McCoy singularities in quantum Ising spin-glasses on d-dimensional hypercubic lattices: A series expansion study

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I will present a study of the $\pm J$ transverse-field Ising spin glass model at zero temperature on d-dimensional hypercubic lattices and in the Sherrington-Kirkpatrick (SK) model, by series expansions around the strong field limit. In the SK model and in high-dimensions our calculated critical properties are in excellent agreement with the exact mean-field results, surprisingly even down to dimension $d = 6$ which is below the expected upper critical dimension of $d=8$. In contrast, in lower dimensions we find a rich singular behavior consisting of critical and Griffiths-McCoy singularities. The divergence of the equal-time structure factor allows us to locate the critical coupling where the correlation length diverges, implying the onset of a thermodynamic phase transition. We find that the spin-glass susceptibility as well as various power-moments of the local susceptibility become singular in the paramagnetic phase before the critical point. Griffiths-McCoy singularities are very strong in two-dimensions but decrease rapidly as the dimension increases. We present evidence that high enough powers of the local susceptibility may become singular at the pure-system critical point.

*Work in collaboration with R.R.P. Singh

Machine Learning of Quantum and Topological Physics

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The machine learning technique can be applied to study a wide range of physical problems including statistical mechanics and various lattice models. Before that, we first apply the machine learning method to known physics problems, and we try to understand how it works and what exact the neural network learns when dealing with physics problems. In the first part of my talk, I will discuss using neural network to classify topological phases. I will discuss that the neural network indeed finds out the right formula for calculating topological invariant after training, with which it can classify topological insulators. The similar scheme can also be applied to the Kosterlitz-Thouless transition. In the second part of my talk, I will discuss using neural network to solve the quantum mechanical scattering problems. After training the neural network, we find that the neural network automatically develops systematical perturbation theory. We shall also discuss a recurrent neural network to predict density directly from potential, which may find its use in the density functional theory.

Decoding quantum criticalities from fermionic/parafermionic topological states

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Under an appropriate symmetric bulk bipartition in a one-dimensional symmetry protected topological (SPT) phase with the Affleck-Kennedy-Lieb-Tasaki matrix product state wave function for the odd integer spin chains, a bulk critical entanglement spectrum can be obtained, describing the excitation spectrum of the critical point separating the SPT phase from the trivial (vacuum) state^{1,2}. Such a critical point is beyond the standard Landau-Ginzburg-Wilson paradigm for symmetry breaking phase transitions. Recently, the general framework of matrix product states for topological phases with Majorana fermions/parafermions have been established^{3,4}.

In this talk, the generalized matrix product states are discussed to describe one-dimensional topological phases of fermions/parafermions in the Fock representation. The defining feature of these topological phases is the presence of Majorana/parafermion zero modes localized at the edges. It is shown that the single-block bipartite entanglement spectrum and its entanglement Hamiltonian are described by the effective coupling between two edge quasiparticles⁵. Furthermore, we demonstrate that sublattice bulk bipartition can create an extensive number of edge quasiparticles in the reduced subsystem, and the symmetric couplings between the nearest neighbor edge quasiparticles lead to the critical entanglement spectra, characterizing the topological phase transitions from the fermionic/parafermionic topological phases to its adjacent trivial phase⁵. The corresponding entanglement Hamiltonians for the critical entanglement spectra can also be derived.

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Born Machine: Unsupervised Generative Modeling using Matrix Product States

Pan Zhang

Generative modeling, which learns joint probability distribution from training data and generates samples according to it, is an important task in machine learning and artificial intelligence. Inspired by probabilistic interpretation of quantum physics, we propose a generative model using matrix product states, which is a tensor network originally proposed for describing (particularly one-dimensional) entangled quantum states. Our model enjoys efficient learning by utilizing the density matrix renormalization group method which allows dynamic adjusting dimensions of the tensors, and offers an efficient direct sampling approach, Zipper, for generative tasks. We apply our method to generative modeling of several standard datasets including the principled Bars and Stripes, random binary patterns and the MNIST handwritten digits, to illustrate ability of our model, and discuss features as well as drawbacks of our model over popular generative models such as Hopfield model, Boltzmann machines and generative adversarial networks. Our work shed light on many interesting directions for future exploration on the development of quantum-inspired algorithms for unsupervised machine learning, which is of possibility of being realized by a quantum device.

Landau-Lifshitz-Gilbert equation and phase transitions

Bo Zheng

The Landau-Lifshitz-Gilbert equation is numerically solved at zero and finite temperature.

The order-disorder phase transition and dynamic pinning-depinning phase transition are then simulated based on the non-equilibrium dynamic approach.

Percolation, Frustration, and Computational Complexity

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The ground states (optimal solutions) of an Ising spin-glass system or a Ising combinatorial optimization problem may be degenerate. If this happens, then some vertices will have their spins changing among different ground-state configurations. These unfrozen vertices may form a giant connected component. In this percolating case, long-range frustrations may exist among the unfrozen vertices in the sense that certain combinations of spin values for these vertices may never appear in any ground-state configuration. We present a mean field theory to tackle such long-range frustrations and apply it to the NP-hard minimum vertex-cover (hard-core gas condensation) problem. Our analytical results on the ground-state energy density and on the fraction of frozen vertices are in good agreement with known numerical and mathematical results. We will discuss the relation between long-range frustration and intrinsic computational complexity.

Enhanced sampling and metastable state analyzing in soft matters

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Soft matter, such as polymers, solution of proteins or DNAs, have very rugged potential energy surface in their conformational spaces. The simulations of equilibrium properties and dynamical behaviors are usually difficult due to too long required simulation time. We develop some simulation techniques, such as temperature walking on generalized canonical ensemble, re-weighting ensemble dynamics simulations, for enhanced sampling the systems. A trajectory-map algorithm, which provides a direct network representation in high-dimensional conformational space based on meta-stable states and interstate transitions, is also presented to accelerate the molecular dynamics simulations and to understand results of simulations. We illustrate the application of the techniques in Lattice spin models, Lennard-Jones fluids, and polypeptide.

Finite-size scaling above the upper critical dimension

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Finite-size scaling (FSS) is a fundamental physical theory within statistical mechanics, describing the asymptotic approach to the thermodynamic limit of finite systems in the neighbourhood of a critical phase transition. It is well-known that models of critical phenomena typically possess an upper critical dimension, d_c , such that in dimensions $d > d_c$, their thermodynamic behaviour is governed by critical exponents taking simple mean-field values. In contrast to the simplicity of the thermodynamic behaviour, the theory of FSS in dimensions above d_c is surprisingly subtle and remains the subject of ongoing debate. In this talk, we will introduce a random walk model on finite graphs with randomly chosen walk length. We will prove a simple connection between the scaling of the mean walk length and the Green's function, and numerically show that our model exhibits the same FSS behaviour as the Ising model and the self-avoiding walk above d_c .

Keywords: Finite-size scaling, upper critical dimension

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Universal state equation of quasi-particle gas for American domestic passenger flights

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Flight delays happen every day in airports all over the world, leading to economic losses of airlines, air traffic disorder and downgrade travelling quality of many people. Up till today, big data of arrival and departure records have been accumulated by air traffic administration agencies. However, general law of the collective motion of passenger flights based on big data and statistical physics is still expected. We model domestic passenger flights of America from 1995 to 2014 as quasi-particle gases, define dimensionless velocity to include the effects from all disturbing factors to passenger flights, and define quasi-thermodynamic quantities to set up the state equations for the gases. By fitting a van der Waals – like state equation to real data, we verify the existence of a universal gaseous constant R , and find the competition effect between attraction from the air routes and that from airports. Furthermore, we set up an exact state equation in the spirit of mean-field theory. Intensive quantity a'_2 is found to have positive linear correlation with average delay per flight in the airports. Most importantly, a universal form of state equations is found based on the principle of correspondence states with the numerical solutions of the critical point.

Critical percolation in anisotropic systems

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When different bonds in a percolating system have different probabilities, the clusters have the tendency to grow at different rates in different directions. For example, when the horizontal bond probability p_h is greater than the vertical bond probability $p_v = 1 - p_h$ on a square lattice, the cluster will grow more in the horizontal direction than in the vertical direction. If you readjust the geometric representation of the lattice so that the horizontal bonds have a length of l_h and the vertical bonds have length l_v , where $l_h/l_v = \tan(3/2 \arctan [\sqrt{3}(1-p_h)/(1+p_h)])$, then the clusters will be isotropic. We generalize this for the checkerboard lattice where now the geometric embedding becomes a parallelogram with a tilt. We verify that shape-dependent properties, such as wrapping probability and excess number for a periodic system and crossing probability for an open system, are as expected for systems of the effective shape. Finally, we conjecture that the Temperley-Lieb formula for the number of clusters per site can be generalized for the checkerboard lattice, and find good numerical confirmation that this is correct.

